ENERGY PRINCIPLE

First Law of Thermodynamics

For a given system

$$\Delta E = Q - W$$

$$\frac{dE}{dt} = \dot{Q} - \dot{W}$$

Where:

$Q = Heat\ transferred\ to\ the\ system\ (poistive\ sign)$

$W = Work\ transferred\ out\ of\ the\ system\ (negative\ sign)$

$E = Energy\ of\ the\ system\ (Kinetic\ Energy, Potential\ Energy, Chemical\ Energy, Electrical\ Energy)$

$E = U + K.E + P.E$

ENERGY PRINCIPLE

Derivation of Energy Equation

Using the Reynolds transport theorem which is

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{cv} b\rho dQ + \int_{cs} b\rho V \bullet dA$$

$$B_{sys} = (E)_{sys} = (mu + me_K + me_P)_{sys}$$

$$b = e = u + e_K + e_P$$

$$\frac{d(E)_{sys}}{dt} = \frac{d}{dt}\int_{cv} e\rho dQ + \int_{cs} e\rho V \bullet dA$$

$$\dot{Q} - \dot{W} = \frac{d}{dt}\int_{cv} e\rho dQ + \int_{cs} e\rho V \bullet dA$$

ENERGY PRINCIPLE

$$\dot{Q} - \dot{W} = \frac{d}{dt}\int_{cv}(u + e_K + e_P)\rho dQ + \int_{cs}(u + e_K + e_P)\rho V \bullet dA$$

$$e_K = \frac{(1/2)mv^2}{m} = \frac{v^2}{2}$$

$$e_P = \frac{mgz}{m} = gz$$

$$\dot{Q} - \dot{W} = \frac{d}{dt}\int_{cv}(u + \frac{v^2}{2} + gz)\rho dQ + \int_{cs}(u + \frac{v^2}{2} + gz)\rho V \bullet dA$$

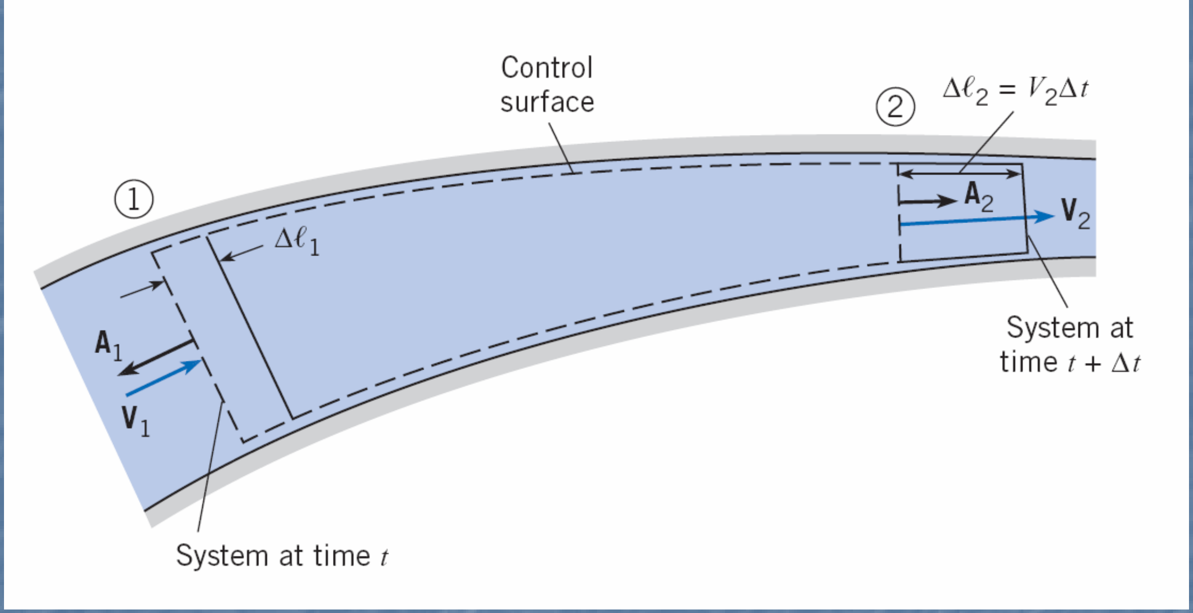
$$\text{The Work } \dot{W} = \dot{W}_f + \dot{W}_s$$

$$\dot{W}_s = \text{Shaft Work}$$

$$\dot{W}_f = \text{Flow Work}$$

ENERGY PRINCIPLE

Flow Work



The flow of work done by the system at station (2) is given by

$$W_{f2} = F \times \Delta l_2 = (p_2 A_2) \times v_2 \Delta t$$

ENERGY PRINCIPLE

$$\dot{W}_{f2} = (p_2 A_2) \times v_2 = p_2 v \bullet A$$

The flow of work by the system at station (1) is given by

$$W_{f1} = F \times \Delta l_1 = (p_1 A_1) \times V_1 \Delta t$$

$$\dot{W}_{f1} = (p_1 A_1) \times V_1 = p_1 V \bullet A$$

In general form, the work done by the system for a constant velocity

$$\dot{W}_f = \sum_{cs} pV \bullet A$$

ENERGY PRINCIPLE

In general form, the work done by the system for a variable velocity

$$\dot{W}_f = \int_{cs} pV \bullet dA = \int_{cs} \frac{p}{\rho}(\rho V \bullet dA)$$

Shaft Work $(\dot{W}_s)$

Consider Eqn.

$$\dot{Q} - \dot{W} = \frac{d}{dt}\int_{cv}(u + \frac{v^2}{2} + gz)\rho dQ + \int_{cs}(u + \frac{v^2}{2} + gz)\rho V \bullet dA$$

$$\dot{Q} - \dot{W}_s - \dot{W}_f = \frac{d}{dt}\int_{cv}(u + \frac{v^2}{2} + gz)\rho dQ + \int_{cs}(u + \frac{v^2}{2} + gz)\rho V \bullet dA$$

ENERGY PRINCIPLE

$$\dot{W}_f = \int_{CS} pV \bullet dA = \int_{CS} \frac{p}{\rho}\left(\rho V \bullet dA\right)$$

$$\dot{Q} - \dot{W}_s - \int_{CS} \frac{p}{\rho}\left(\rho v \bullet dA\right) = \frac{d}{dt}\int_{cv} (u + \frac{v^2}{2} + gz)\rho dQ + \int_{cs} (u + \frac{v^2}{2} + gz)\rho V \bullet dA$$

$$\dot{Q} - \dot{W}_s = \frac{d}{dt}\int_{CV} (u + \frac{v^2}{2} + gz)\rho dQ + \int_{CS} (u + \frac{v^2}{2} + gz)\rho V \bullet dA + \int_{CS} \frac{p}{\rho}\left(\rho V \bullet dA\right)$$

$$\dot{Q} - \dot{W}_s = \frac{d}{dt}\int_{CV} (u + \frac{v^2}{2} + gz)\rho dQ + \int_{CS} (u + \frac{p}{\rho}) + \frac{v^2}{2} + gz)\rho V \bullet dA$$

$$\text{The term enthalpy} = h = \left(u + \frac{p}{\rho}\right)$$

$$\dot{Q} - \dot{W}_s = \frac{d}{dt}\int_{CV} (u + \frac{v^2}{2} + gz)\rho dQ + \int_{CS} (h + \frac{v^2}{2} + gz)\rho V \bullet dA$$

ENERGY PRINCIPLE

Steady Flow Energy Equation

For steady flow, the flow accumulation equal zero and if the **velocity distribution is constant**, then

the Eqn. $\dot{Q} - \dot{W}_s = \frac{d}{dt}\int_{CV}(u + \frac{v^2}{2} + gz)\rho dQ + \int_{CS}(h + \frac{v^2}{2} + gz)\rho V \bullet dA$ becomes

$$\dot{Q} - \dot{W}_S = \sum_{CS} \dot{m}_{out}\left(\frac{V^2}{2} + gz + h\right)_{out} - \sum_{CS} \dot{m}_{in}\left(\frac{V^2}{2} + gz + h\right)_{in}$$

ENERGY PRINCIPLE

The coefficients (α_1, α_2) *are called kinetic energy correction factor and can be evaluated as follows:*

$$\alpha \frac{\rho \bar{V}^3 A}{2} = \int_A \frac{\rho V^3 dA}{2}$$

$$\alpha = \frac{1}{A} \int_A \left(\frac{V}{\bar{V}} \right)^3 dA$$

$\alpha = 1$ *Velocity is uniform*

$\alpha > 1$ *Velocity is non-uniform*

$\alpha = 2$ *Laminar Flow*

$\alpha = 1.05$ *Turbulent Flow*

ENERGY PRINCIPLE

Pump – Turbine Combination

When a compressor and a turbine are connected together, the resultant shaft work $\dot{W}_S$ **is equal to**

$$\dot{W}_S = \dot{W}_T - \dot{W}_p$$

Substituting for the above expression in Eqn. below

$$\frac{1}{\dot{m}}(\dot{Q} - \dot{W}_s) + \left(\frac{p_1}{\rho} + gz_1 + u_1 + \alpha_1 \frac{\overline{V_1}^2}{2}\right) = \left(\frac{p_2}{\rho} + gz_2 + u_2 + \alpha_2 \frac{\overline{V_2}^2}{2}\right)$$

and divide by (g), we have

$$\frac{\dot{Q}}{\dot{m}g} - \left(\frac{\dot{W}_T - \dot{W}_P}{g}\right) + \left(\frac{p_1}{\rho g} + z_1 + \frac{u_1}{g} + \alpha_1 \frac{\overline{V_1}^2}{2g}\right) = \left(\frac{p_2}{\rho g} + gz_2 + \frac{u_2}{g} + \alpha_2 \frac{\overline{V_2}^2}{2g}\right)$$

ENERGY PRINCIPLE

Rearranging, we have

$$\left(\frac{\dot{W}_p}{\dot{m}g}\right)+\left(\frac{p_1}{\rho g}+z_1+\alpha_1\frac{\overline{V}_1^{\,2}}{2g}\right)=\left(\frac{\dot{W}_T}{\dot{m}g}\right)+\left(\frac{p_2}{\rho g}+gz_2+\alpha_2\frac{\overline{V}_2^{\,2}}{2g}\right)+\left(\frac{u_2-u_1}{g}\right)-\left(\frac{\dot{Q}}{\dot{m}g}\right)$$

$$\left(h_P+\frac{p_1}{\rho g}+z_1+\alpha_1\frac{\overline{V}_1^{\,2}}{2g}\right)_{Mech.part}=\left(h_T+\frac{p_2}{\rho g}+gz_2+\alpha_2\frac{\overline{V}_2^{\,2}}{2g}\right)_{Mech.part}+\left(\frac{u_2-u_1}{g}-\frac{\dot{Q}}{\dot{m}g}\right)_{thermal\ part}$$

$$h_P=\frac{\dot{W}_C}{\dot{m}g}=Pump\ Head$$

$$h_T=\frac{\dot{W}_T}{\dot{m}g}=Turbine\ Head$$

$$The\ Head\ Loss=h_{Loss}=\left(\frac{1}{g}\right)\left(u_2-u_1-\frac{\dot{Q}}{\dot{m}}\right)$$

$$\left(h_P+\frac{p_1}{\rho g}+z_1+\alpha_1\frac{\overline{V}_1^{\,2}}{2g}\right)_{Mech.part}=\left(h_T+\frac{p_2}{\rho g}+gz_2+\alpha_2\frac{\overline{V}_2^{\,2}}{2g}\right)_{Mech.part}+h_{Loss}$$

ENERGY PRINCIPLE

$$Pump\ Head = h_C = \frac{\dot{W}_C}{\dot{m}g}$$

$$\dot{W}_C = \dot{m}gh_C = \gamma Q h_C$$

$$\dot{W}_T = \dot{m}gh_T = \gamma Q h_T$$

$$Pump\ Efficiency = \eta_P = \frac{(\dot{W}_P)}{(\dot{W}_P)_{actual}}$$

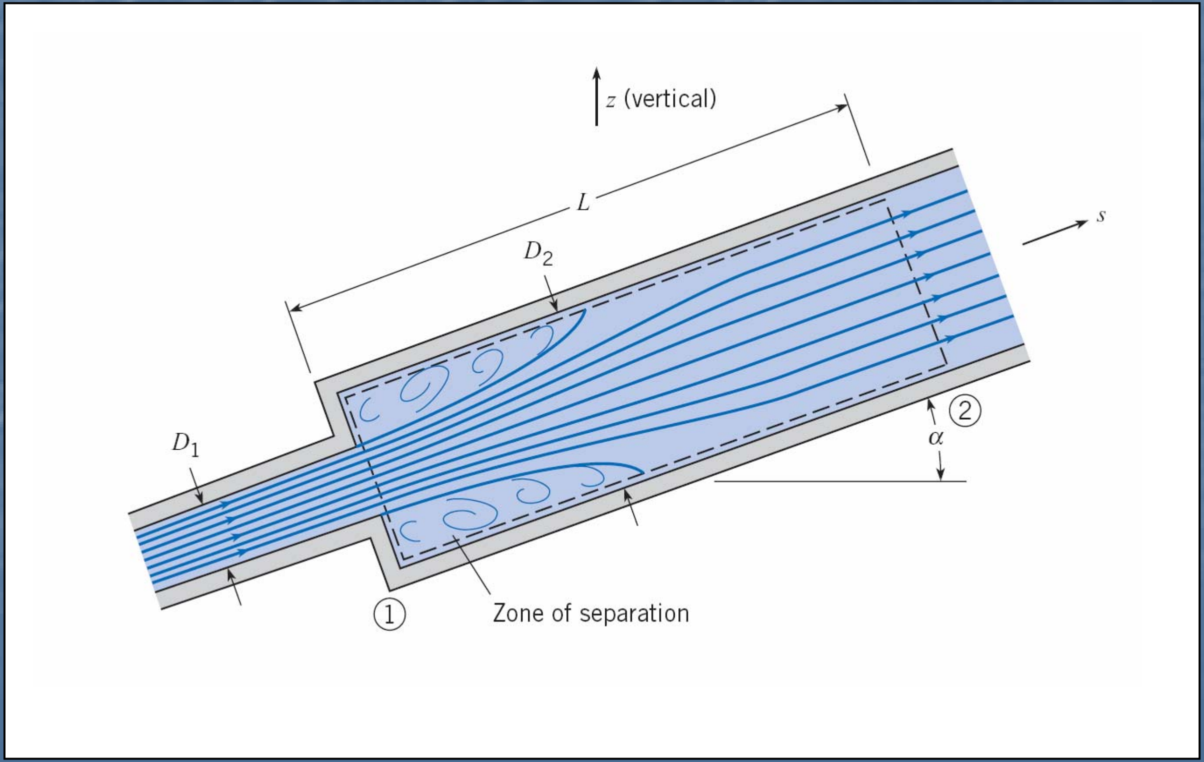
$$Turbine\ Efficiency = \eta_T = \frac{(\dot{W}_T)_{actual}}{(\dot{W}_T)}$$

ENERGY PRINCIPLE

Application of the Energy Equation, Momentum and Continuity Principles in Combination

Abrupt Expansion

$$i.e.\quad h_{loss} = \left(\frac{V_1^2 - V_2^2}{2g}\right)$$



THE END